Tree Diagrams and the Fundamental Counting Principle

The purpose of this task is to help students discover the Fundamental Counting Principle through the use of tree diagrams. Additionally, the tree diagrams are used to solve problems related to cost and probability.

As an introduction to this activity, situations could be given to small groups for them to prepare tree diagrams and then compare the number of possible outcomes for each situation. Small groups could share their tree diagrams and the outcomes. By listing the situations and the possible outcomes students should be able to determine the relationship between the number of decisions and the number of outcomes.

Situations for Group Practice:
Assign one of the following situations to each group. Give each group a piece of paper and marker. Each group should create a tree diagram to determine the number of outcomes.

1. Flip a dime and then flip a quarter
2. A choice of chicken, fish or beef for the main dish and a choice of cake or pudding for dessert
3. A choice of either a green or blue shirt and a choice of blue, black or khaki pants
4. A choice of pizza or spaghetti; a choice of milk or juice to drink; a choice of pudding or an apple for dessert
5. Shirts come on three sizes: small, medium or large; shirts have buttons or snaps; colors are blue or beige
6. The choices for school mascot are lion, bear and porpoise; colors are red, blue and gold

As students present their work create a table like the one below:

Decisions	Possible Outcomes
2, 2	4
3, 2	6
2, 4	8
2, 2, 2	8
3, 3	9
3, 2, 2	12

Ask: Is there a relationship between the number of decisions to be made and the possible outcomes? *The possible outcomes is the product of the decisions – this is the **Fundamental Counting Principle***

The **Fundamental Counting Principle** tells us that if we have two decisions to make, and there are *M* ways to make the first decision, and *N* ways to make the second decision, the product of *M* and *N* tells us how many different outcomes there are for the overall decision process. In general, when a series of decision are to be made, the product of all the way to make the individual decisions determines the number of outcomes there are.

The following pages offer two scenarios for students to solve probability problems using tree diagrams.

Name: _____ Date: _____

Travel Time

A travel agent plans trips for tourists from Chicago to Miami. He gives them three ways to get from town to town: airplane, bus, train. Once the tourists arrive, there are two ways to get to the hotel: hotel van or taxi. The cost of each type of transportation is given in the table below.

Transportation Type	Cost
Airplane	$350
Bus	$150
Train	$225
Hotel Van	$60
Taxi	$40

1. Draw a tree diagram to illustrate the possible choices for the tourists. Determine the cost for each outcome.

2. If these six outcomes are chosen equally by tourists, what is the probability that a randomly selected tourist travel in a bus?

3. What is the probability that a person's trip cost less than $300?

4. What is the probability that a person's trip costs more than $350?

5. If the tourists were flying to New York, the subway would be a third way to get to the hotel. How would this change the number of outcomes? Use the Fundamental Counting Principle to explain your answer.

Name: _____ Date: _____

"Happy Birthday to You"

Andy has asked his girlfriend to make all the decisions for their date on her birthday. She will pick a restaurant and an activity for the date. Andy will choose a gift for her. The local restaurants include Mexican, Chinese, Seafood, and Italian. The activities she can choose from are Putt-Putt, bowling, and movies. Andy will buy her either candy or flowers.

1. How many outcomes are there for these three decisions? _____
2. Draw a tree diagram to illustrate the choices.

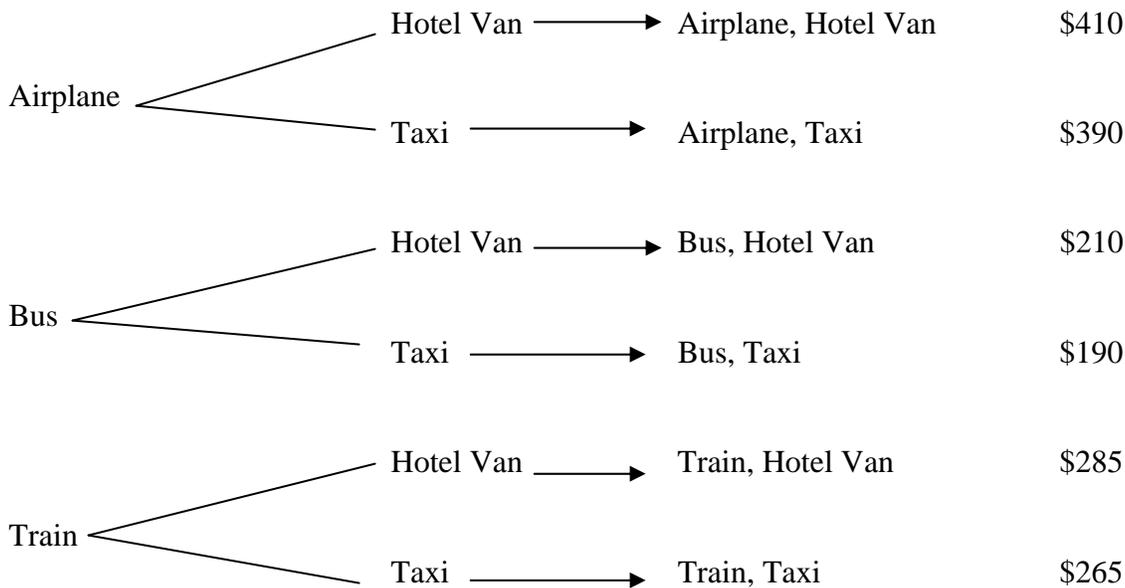| Dinner for Two	Activity Cost for Two	Gift Cost
Mexican - $20	Putt-Putt - $14	Flowers - $25
Chinese - $25	Bowling - $10	Candy - $7
Italian - $15	Movies - $20	

3. If all the possible outcomes are equally likely, what is the probability that the date will cost at least $50?

4. What is the maximum cost for the date?

5. What is the minimum cost for the date?

6. To the nearest dollar, what is the average cost for this date?

7. What is the probability that the date costs exactly $60?

8. What is the probability that the date costs under $40?

Travel Time Answer Key

A travel agent plans trips for tourists from Chicago to Miami. He gives them three ways to get from town to town: airplane, bus, train. Once the tourists arrive, there are two ways to get to the hotel: hotel van or taxi. The cost of each type of transportation is given in the table below.

Transportation Type	Cost
Airplane	$350
Bus	$150
Train	$225
Hotel Van	$60
Taxi	$40

1. Draw a tree diagram to illustrate the possible choices for the tourists. Determine the cost for each outcome.

Airplane	Hotel Van →	Airplane, Hotel Van	$410
	Taxi →	Airplane, Taxi	$390
Bus	Hotel Van →	Bus, Hotel Van	$210
	Taxi →	Bus, Taxi	$190
Train	Hotel Van →	Train, Hotel Van	$285
	Taxi →	Train, Taxi	$265

2. If these six outcomes are chosen equally by tourists, what is the probability that a randomly selected tourist travel in a bus? $\frac{2}{6}$ or $\frac{1}{3}$

3. What is the probability that a person's trip cost less than $300? $\frac{3}{6}$ or $\frac{1}{2}$

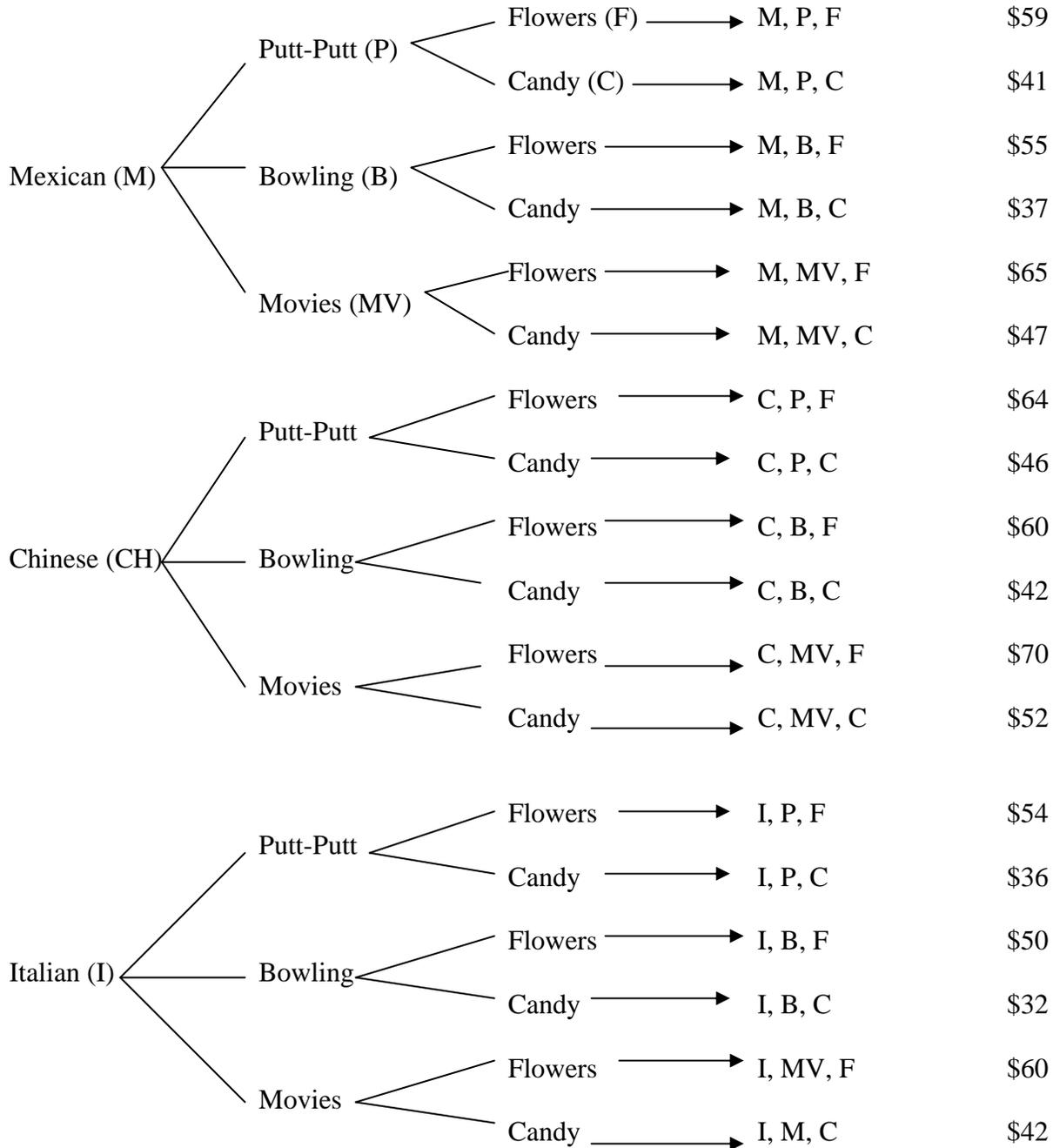4. What is the probability that a person's trip costs more than $350? $\frac{2}{6}$ or $\frac{1}{3}$

5. If the tourists were flying to New York, the subway would be a third way to get to the hotel. How would this change the number of outcomes? Use the Fundamental Counting Principle to explain your answer. *Using the Fundamental Counting Principle, I would multiply 3 x 3 to get 9 outcomes.*

"Happy Birthday to You" Answer Key

Andy has asked his girlfriend to make all the decisions for their date on her birthday. She will pick a restaurant and an activity for the date. Andy will choose a gift for her. The local restaurants include Mexican, Chinese, Seafood, and Italian. The activities she can choose from are Putt-Putt, bowling, and movies. Andy will buy her either candy or flowers.

1. How many outcomes are there for these three decisions? __18__
2. Draw a tree diagram to illustrate the choices.

Mexican (M)			
Putt-Putt (P)	Flowers (F) → M, P, F	$59
	Candy (C) → M, P, C	$41
Bowling (B)	Flowers → M, B, F	$55
	Candy → M, B, C	$37
Movies (MV)	Flowers → M, MV, F	$65
	Candy → M, MV, C	$47

Chinese (CH)		
Putt-Putt	Flowers → C, P, F	$64
	Candy → C, P, C	$46
Bowling	Flowers → C, B, F	$60
	Candy → C, B, C	$42
Movies	Flowers → C, MV, F	$70
	Candy → C, MV, C	$52

Italian (I)		
Putt-Putt	Flowers → I, P, F	$54
	Candy → I, P, C	$36
Bowling	Flowers → I, B, F	$50
	Candy → I, B, C	$32
Movies	Flowers → I, MV, F	$60
	Candy → I, M, C	$42

"Happy Birthday to You" Answer Key (continued)

Dinner for Two	Activity Cost for Two	Gift Cost
Mexican - $20	Putt-Putt - $14	Flowers - $25
Chinese - $25	Bowling - $10	Candy - $7
Italian - $15	Movies - $20	

3. If all the possible outcomes are equally likely, what is the probability that the date will cost at least $50? $\dfrac{10}{18}$ or $\dfrac{5}{9}$

4. What is the maximum cost for the date? $70

5. What is the minimum cost for the date? $32

6. To the nearest dollar what is the average cost for this date? $51

7. What is the probability that the date costs exactly $60? $\dfrac{2}{18}$ or $\dfrac{1}{9}$

8. What is the probability that the date costs under $40? $\dfrac{3}{18}$ or $\dfrac{1}{6}$